Looking for Slope in All the Wrong Places

Photograph 1
Road sign, Provincetown, Massachusetts

RON LANCASTER

"Mathematical Lens" uses photographs as a springboard for mathematical inquiry and appears in every issue of the *Mathematics Teacher*. All submissions should be sent to the department editors. For more background information on "Mathematical Lens" and guidelines for submitting a photograph and questions, please visit http://www.nctm.org/publications/content.aspx?id=10440#lens.

Edited by ***Ron Lancaster***
ron2718@nas.net
University of Toronto
Toronto, Ontario M5S 1A1
Canada

Brigitte Bentele
brigitte.bentele@trinityschoolnyc.org
Trinity School
New York, NY 10024

1. (*a*) Estimate, without doing any calculations, which of the following fractions is closest to the slope of the hill depicted in the road sign in Provincetown, Massachusetts, shown in **photograph 1**:

$$0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{4}, \frac{3}{4}, \frac{1}{3}, \frac{2}{3}, \frac{1}{2}, 1$$

(*b*) Estimate, without doing any calculations, which of the following series of angles is closest to the three angles of the triangle shown in the road sign in **photograph 1**:

$5°, 85°, 90°$
$15°, 75°, 90°$
$25°, 65°, 90°$
$35°, 55°, 90°$
$45°, 45°, 90°$

(*c*) Measure the legs of the triangle shown in the sign in **photograph 1** and the angles opposite them. You can do this directly on the photograph by using a ruler and a protractor, or you can paste the image into a file in The Geometer's Sketchpad™ (GSP) and use the available tools to make the measurements. (Go to www.nctm.org for a downloadable electronic file.) Compare these measurements with your estimates from parts (*a*) and (*b*).

(*d*) Let m be the slope of segment AC in $\triangle ABC$ with $m\angle ABC = 90°$ (see **fig. 1**). Assume that the right angle remains fixed but that the other angles change as the slope of $\overline{AC}$ changes. Match each of the

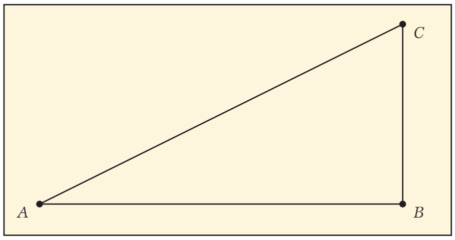

Fig. 1 What happens to the acute angles in $\triangle ABC$ as the slope of segment AC changes?

following with the appropriate graph in **figure 2**:

(i) The graph of m versus $\angle CAB$

(ii) The graph of m versus $\angle BCA$

(iii) The graph of m versus $\angle ABC$

2. As you may have realized, the symbol of the hill shown in the sign in **photograph 1** does not provide any information about the actual steepness of the hill. The sign is a generic sign that could be posted on any hill. In contrast, **photographs 2** and **3**, taken in Hong Kong, show a method used in other countries to provide drivers with information about the steepness of a hill.

   (*a*) What is the connection between the ratios shown in **photographs 2** and **3** and the slopes of the respective hills?

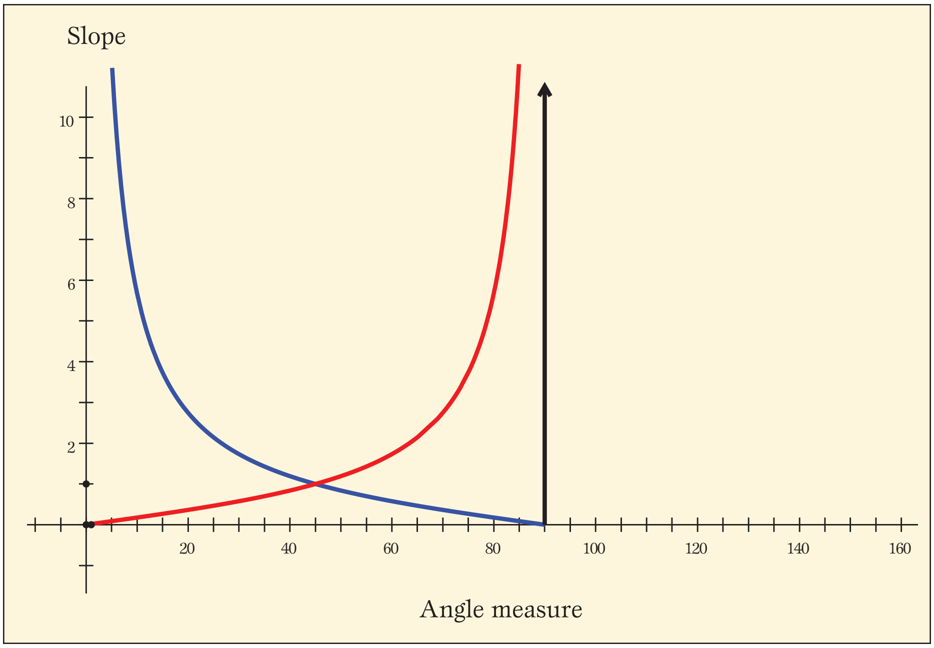

Fig. 2 The blue, red, and black graphs can be associated with each of the situations.

   (*b*) The second term of the ratio shown in the sign in **photograph 2** is larger than the second term of the ratio shown in the sign in **photograph 3**. Does this mean that the hill in **photograph 2** is steeper than the hill in **photograph 3**?

   (*c*) In addition to ratios, what other methods can be used to inform drivers of the actual steepness of a hill?

3. You may have noticed that the orientation of the hill in the Provincetown sign is the reverse of the orientation of the hills in the Hong Kong signs. In both symbols the hill descends, with the bicycle facing downhill and the road leading down. Yet the symbols do not have slopes of the same sign. The hill symbol in the photograph taken in the United States has a positive slope, whereas the hill symbols in the photographs taken in Hong Kong have a negative slope. Via the Internet, explore road signs in various countries to provide a possible explanation.

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Photograph 2 Road sign, Hong Kong

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Photograph 3 Road sign, Hong Kong

1. (a) 1/2.

   (b) 25°, 65°, 90°.

   (c) The editors pasted **photograph 1** into a GSP file (see **fig. 3**). Using the measuring tools, we found the rise and the run to be approximately 1.34 cm and 2.70 cm, respectively, and the angles opposite these sides to be about 26.37° and 63.63°, respectively. Therefore, the slope of the hill in photograph 1 is about 1/2. If the approximations for parts (a) and (b) were done carefully, the results for parts (a), (b), and (c) should be consistent with one another.

   (d) Because the slope of segment AC increases when $m\angle CAB$ increases from 0° to 90°, it follows that the graph of m versus $\angle CAB$ is the red graph shown in **figure 2**. In contrast, the slope of $\overline{AC}$ decreases when $m\angle BCA$ increases from 0° to 90°, so the graph of m versus $\angle BCA$ is the blue graph in **figure 2**. The $m\angle ABC$ is always equal to 90° and has no bearing on the slope of $\overline{AC}$. Thus, the graph of m versus $\angle ABC$ is the black graph in **figure 2**.

   Students who have studied trigonometry can match the graphs by letting $x = \angle CAB$ and noting that $y = \tan x$ is the equation for the graph of m versus $\angle CAB$. Therefore, $y = \tan(90° - x)$ is the equation for the graph of m versus $\angle BCA$.

Fig. 3 Importing the photograph into GSP makes it easy to determine the slope of the hill symbol depicted in the sign.

2. (a) The ratio 1:8 indicates that the slope of the hill is 1/8, and the ratio 1:5 tells drivers that the slope of the hill is 1/5.

   (b) Because 1/8 < 1/5, the hill in **photograph 2** is not steeper than the hill in **photograph 3**. If the first term (the numerator) of the ratio is 1, then the slope of the hill varies inversely with the second term (the denominator) of the ratio.

   (c) Percentages can also be used to inform drivers of the steepness of a hill. Readers are invited to submit a photograph of an example of this method to the "Mathematical Lens" editors for possible publication in a future issue of the *Mathematics Teacher*.

3. Please send your findings to the "Mathematical Lens" editors for possible publication in a future *MT* issue.

For a mathematical photograph for which you may create your own questions, go to the NCTM Web site: www.nctm.org/mt. Send your questions to the "Mathematical Lens" editors.

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Use this photograph to create your own questions in the style of “Mathematical Lens.” Send your questions to the “Mathematical Lens” editors: Ron Lancaster, ron2718@nas.net, or Brigitte Bentele, brigitte.bentele@trinityschoolnyc.org.

Photograph taken by Ron Lancaster at Brock University in St. Catharines, Ontario.